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RELATIVISTIC MODELS OF THE STRUCTURE OF LIGHT NUCLEI

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In this talk I will discuss how relativistic meson theory is developed and applied to the description of few nucleon systems, and show how the ideas which emerge from this discussion provide a natural, theoretical foundation for the Dirac treatment of proton-nucleus scattering.

There are several different ways to develop the relativistic description of few body systems, and four of these methods, all based on the consideration of how to sum an infinite set of diagrams, will be discussed briefly in Section 1. I will then digress briefly, and show in Section 2 how one of these methods provides a natural justification for the popular Dirac treatment of proton-nucleus scattering. Section 3 will return to the few body system and summarize some applications of the relativistic few body equations derived in Section 1, including a brief account of some recent fits to the nucleon-nucleon phase shifts not yet published.

What is relativistic meson theory, and what role can it have in nuclear physics now that we know about quarks and QCD? I view it as a consistent relativistic theory of effective interactions between selected quark clusters, which are treated as structureless particles. The emphasis is on the words "consistent" and "relativistic." This means that we will insist that the theory be manifestly covariant at every step (although I will not hesitate to do calculations in especially convenient frames of reference), and that all interaction operators involving external probes be consistent with the relativistic "potential" V (which is actually the kernel of an integral equation). Some attempt is made to allow for the structure of the mesons and nucleons by inserting phenomenological form factors at the vertices and, in some cases, using simple phenomenological functions for self energies, but the basic equations of the theory are obtained from a Lagrangian for point-like mesons and nucleons. The justification for using such a theory today is that it gives a calculable theory of nuclei which employs the degrees of freedom most apparent in nuclear physics, and which through detailed comparison with experiment can help us uncover those phenomena which require the explicit use of quark degrees of freedom.

1. RELATIVISTIC WAVE EQUATIONS

1.1 Types of Equations

Relativistic equations can be written in the following very general form

$$\mathcal{M} = \mathcal{V} + \mathcal{V}G\mathcal{M} \tag{1}$$

where $\mathcal{M}$ is the scattering amplitude, $\mathcal{V}$ is the kernel or relativistic potential, and G the propagator. If $\mathcal{V}$ is in some sense small Eq. (1) can be solved by iteration as shown diagrammatically in Fig. 1 for two particles. We see that the equation can be regarded as a means of summing a generalized Born series, or summing an infinite number of diagrams. If $\mathcal{V}$ is small, the solution to (1) will not differ significantly from taking $\mathcal{V}$ alone, and the equation is not doing much for us. However, when $\mathcal{V}$ is large, the Born series will not exist, but the solution to (1) will. In this sense relativistic equations enable us to treat non-perturbative problems.

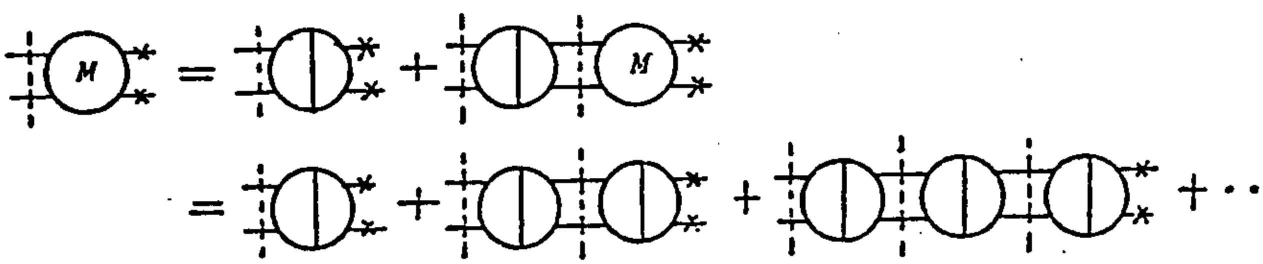


FIGURE 1

Bound state wave functions can be obtained from the residues of the bound state poles of $\mathcal{M}$. Near the bound state pole at M_B,

$$\mathcal{M}\,(p\ p',P) = \frac{\Gamma(p)\Gamma^{+}(p')}{M_B^2 - P^2} + \mathcal{R} \tag{2}$$

where p and p' are the relative 4 momenta of the final and initial state respectively, P is the total 4 momentum, and $\mathcal{R}$ is a remainder function regular at $P^2 = M_B^2$. Substituting (2) into (1) one can obtain both the bound state wave equation and the normalization condition[1]

$$\Gamma = \mathcal{V}G\,\Gamma \tag{3}$$

$$1 = \int \Gamma^{+} \left(\frac{dG}{dP^2} - G\,\frac{d\mathcal{V}}{dP^2}G\right)\Gamma \tag{4}$$

The relativistic wave function ψ is related to the vertex function Γ by

$$\psi = VG\Gamma$$
$$\Gamma = V\psi \qquad (5)$$

To find the relativistic kernel V from an infinite class of diagrams, one must first decide on what class of diagrams to sum, and then introduce a scheme for organizing the sum. I will assume that the smallest class of diagrams which will describe the dynamics adequately is the sum of all ladder and crossed ladder Feynman diagrams (with form factors at the vertices and on the propagators). In particular, it is known that crossed ladders make important contributions, and therefore the ladder sum alone is certainly not adequate. If particle production and inelasticities are to be treated explicitly, a larger class of Feynman diagrams including self energy contributions is almost certainly necessary, but for elastic processes the ladder and crossed ladder sum may be sufficient. This sum, up to 6th order in the coupling constant, is shown in Fig. 2 for the case of two heavy nucleons exchanging a light meson. The ladder diagrams are (a), (b), and (d); all others are crossed ladders.

The way in which this sum is organized now depends on how the two body propagator G is defined. In the most general case, the propagator G is con-

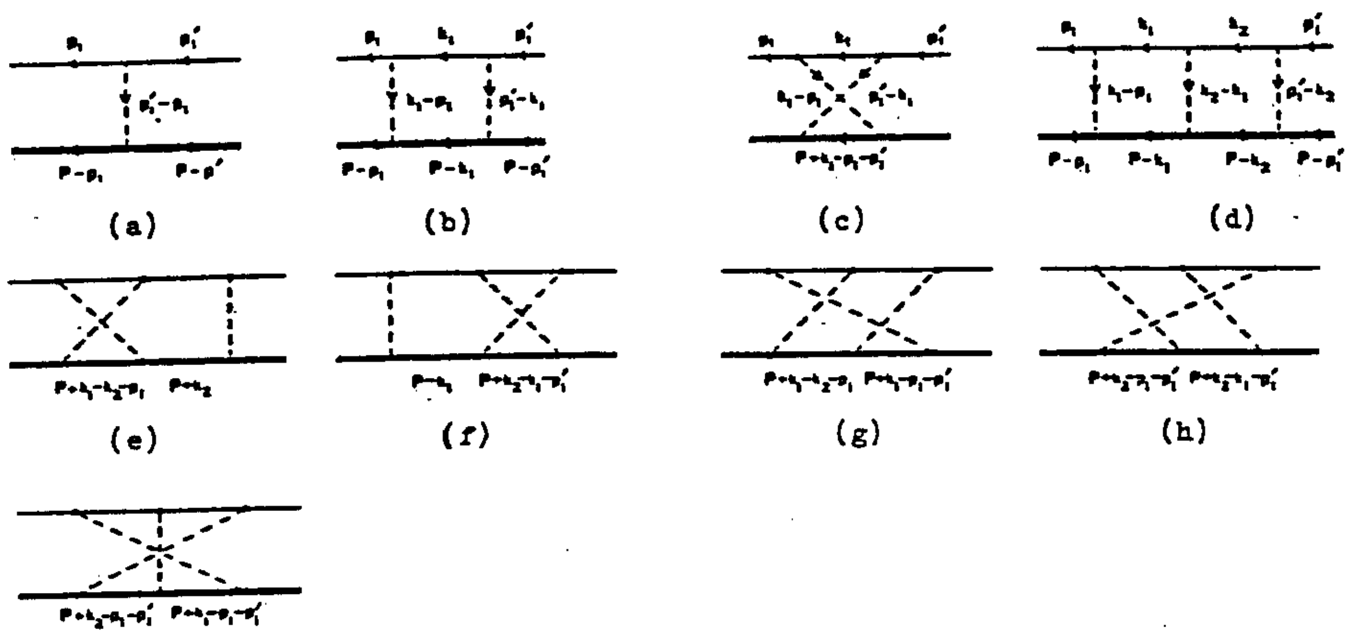


FIGURE 2

strained according to some covariant prescription so that it depends on only the relative 3 momentum instead of the relative 4 momentum. The advantage of such an approach is that the number of free variables is thereby reduced, making the resulting integral equation simpler to solve and easier to interpret. The kernel V corresponding to the constrained G is then the sum of all diagrams which cannot be obtained by iterating lower order kernels as shown in Fig. 1 (where the constrained propagator is represented by a vertical dotted line cutting the two nucleons). Hence the precise definition of V depends on the definition of G. The kernel up to 4th order is shown diagrammatically in Fig. 3. The first diagram (3a) is the one boson exchange (OBE) contribution, the second (3b) is the difference between the full box diagram and the first iteration of the OBE, which is called the subtracted box, and the third (3c) is the crossed box. If the unconstrained 2 body propagator is used, as in the Bethe Salpeter (BS) equation[2], then the full

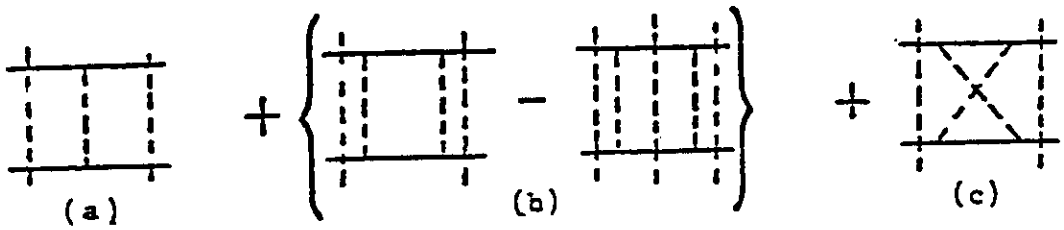


FIGURE 3

box is obtained after one iteration of the OBE, so the subtracted box is zero. With constrained propagators, the full box is not obtained after one iteration, so the subtracted box must be added. In 6th order subtracted boxes and subtracted crossed boxes coming from Figs. 2d-f must be included in the kernel as well as the fully crossed ladder diagrams (2g-i), and so on to all orders. What the relativistic equation has done for us is to replace the full sum in Fig. 2, which certainly does not converge for large coupling constants, with a sum like that shown in Fig. 3 for the kernel. The procedure will only work if the sum for the kernel converges rapidly. Before I discuss this important issue, I will review a number of popular choices for the propagator G.

Four choices of G are summarized in Table I. The BS equation conserves 4 momentum in the intermediate state, so it remains on the energy shell defined by $P_o = W$, where W is the initial energy of the two body system and P_o is the total energy in the intermediate state (both in the CM system). This leaves all four components of the relative 4-momentum, $p = \frac{1}{2}(p_1 - p_2)$, unconstrained. Alternatively, if we restrict one particle to its positive energy mass shell (say particle 2)[3], then $P_o = W$ and $p_{20} = (M^2 + \vec{p}^{\,2})^{1/2} = E_p$ fixes the relative energy in a covariant way

$$p_o = \frac{1}{2}W - E_p$$

leaving only the three components of $\vec{p}$ as free variables. If we wish to restrict both particles to their mass shells, we must drop the requirement that $P_o = W$, or go off the energy shell. One way of doing this was developed by Logunov and Tavkhelidze[4] and by Blankenbecler and Sugar[5]; a variation of this method is due to Todorov[6]. An advantage of this approach is that the number of spin degrees of freedom is reduced because both particles are on shell and hence have only two spin degrees of freedom. Finally, the light front equation[7] comes from a different approach in which field theory is quantized on the light front[8], which loosely speaking refers to quantizing fields at equal values of $\tau = t+x$ (the velocity of light c is taken equal to unity and x can be any one of the three directions in space). The variable

Relativistic Two Body Equations

Name	Description of G	Number of Variables Momentum	Spin
Bethe-Salpeter (BS)	On energy shell Both particles off mass shell	4	$4 \times 4 = 16$
Particle 1 off shell (G_1)	On energy shell One particle off mass shell	3	$2 \times 4 = 8$
Blankenbecler-Sugar Logunov-Tavkhelidze (BSLT)	Off energy shell Both particles on mass shell	3	$2 \times 2 = 4$
Light Front (LF)	Off $p_- = E - p_x$ shell Both particles on mass shell	3	$2 \times 2 = 4$

conjugate to τ is $p_- = E - p_x$ which now plays a role similar to that usually played by the energy, so that this approach bears a close formal resemblance to old fashioned time ordered perturbation theory, where all particles are on the mass shell, but intermediate states are off the energy (p_- in this case) shell. However, while there is a formal resemblance between τ ordered diagrams and time ordered diagrams there is a profound difference which cannot be overemphasized. The τ ordered formalism is manifestly covariant at every step while time ordered perturbation theory breaks covariance. This is related to the fact that τ is invariant under boosts in the x direction, while t is not. A disadvantage of the light front formulation is that it breaks manifest rotational invariance. Several authors have used LF techniques in recent years.(9-12)

The extension of relativistic equations to more than two bodies is a subject of increasing importance. All of the equations mentioned above can be extended but the BS equation has 4 (N-1) integration variables while the constrained equations (in common with non-relativistic equations) have only 3 (N-1). It is important that any n body system with $n<N$ particles must be dynamically independent of the others when the others are beyond the range of forces. A serious deficiency of the BSLT equation is that it does not satisfy this requirement. Namyslowski and Weber(13) have shown that the three body LF equation satisfies the cluster property, and it has been shown(14) that the three body generalization of the G_1 equation also has this property.

I wish to emphasize that the constrained equations should not be regarded as an approximation to the BS equation. From a relativistic point of view, all of the equations are equally good starting points and the question of which equation is "best" will depend on other criteria, such as how rapidly the series for the kernel converges.

1.2 Convergence of the infinite series for V

I now want to discuss the convergence of the infinite series for V, the first three terms of which are shown diagramatically in Fig. 3. If the terms in the $2n^{th}$ order kernel cancel among themselves, making the full $2n^{th}$ order kernel smaller than a typical $2n^{th}$ order term, we conclude that the series for V converges more rapidly than if no cancellations were present.

As an example of how these ideas work, consider the case of a light particle m interacting with a very heavy, neutral scalar particle M. It has been known for many years(15) that in the limit as $M\to\infty$ the ladders and crossed ladders cancel in such a way that the total result can be obtained by iterating the OBE kernel with the heavy particle restricted to its mass shell (other constrained prescriptions also work; when $M \to \infty$ they are equivalent to putting the heavy particle on shell). This means that the irreducible kernel reduces

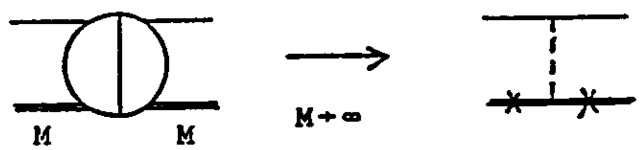


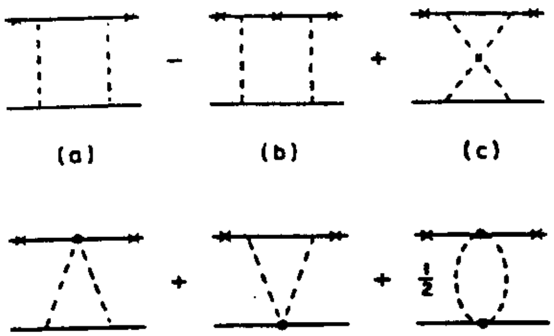
FIGURE 4.

exactly in this limit to OBE as shown in Fig. 4 (there the x means the particle is on-shell). In terms of the diagrams shown in Fig. 3, it means that the subtracted box and the crossed box exactly cancel when $M \to \infty$. Furthermore, the same cancellation takes place in every order, leaving the OBE to give the exact relativistic one body equation for the light particle m (Dirac if m has spin $\frac{1}{2}$, Klein Gordon if m has spin zero) moving in an instantaneous potential.

Unfortunately, the BS equation does not have a one body limit in this sense. In the BS equation, the subtracted box is exactly zero, leaving nothing to cancel the contributions from the crossed box. This happens in every order, so that the BS kernel in the $M \to \infty$ limit remains an infinite sum.

When both particles have spin, or when the heavy one is charged, such general results have not been proved, but may very well be true in some cases. For example, the cancellation has also been observed to work in 4^{th} order for two spin $\frac{1}{2}$ particles in QED.[3,16-17] The cancellation also occurs in 4^{th} order for two heavy spin $\frac{1}{2}$ nucleons exchanging pseudoscalar, isovector pions, provided the π-N interaction is treated in a manner consistent with chiral symmetry.[15] I will now describe these results in somewhat more detail.

To order g^2 in the π-N interaction, chiral invariance implies that the pseudoscalar $g\gamma^5\tau_i$ coupling must be accompanied by a σ-like $2\pi NN$ contact term of the form $g^2\delta_{ij}/M$ where i and j are the isospin indices of the two pions.



(a) (b) (c)

(d) (e) (f)

FIGURE 5

There are then 6 diagrams which contribute to the 4^{th} order kernel as shown in Fig. 5. The first two (a) and (b), are the subtracted box (shown for the case when particle 2 is off shell), (c) is the crossed box, (d) and (e) are triangle diagrams involving one contact interaction, and (f) is the bubble involving two contact interactions. In a scalar φ^3 theory, only the first three occur.

In Ref. (15) the 4^{th} order kernel for both the φ^3 and realistic chiral models were studied for a restricted propagator with a factor

$$\delta_+[(1+\nu)(M^2-k_1^2)-(1-\nu)(M^2-k_2^2)]$$

where $-1\leq\nu\leq 1$ varies so that when $\nu = 1$ particle one is on shell, when $\nu = -1$ particle 2 is on shell, and when $\nu = 0$ they are symmetrically off shell similar to the BSLT case. For the scalar case, in the limit as $M\to\infty$, the result for the subtracted box (4S) and the crossed box (4-) is

$$V_{4S} = V_o - \frac{\mu}{M}[(1-\nu^2) + 2(1+\nu^2)\theta]V_1$$

$$V_{4-} = -V_o + \frac{\mu}{M}[2 + 4\theta]V_1 \tag{6}$$

where μ is the exchanged particle mass,

$$\theta = \frac{1}{\mu^2}(\vec{p}^{\,2} + \vec{p}\,'^2 - 2M\varepsilon) \tag{7}$$

is a non-local, energy dependent term, and

$$V_o(t) = \frac{g^4}{16\pi^2M^2} \int_{4\mu^2}^{\infty} \frac{d\xi}{\xi-t}\{\xi[\xi-4\mu^2]\}^{-1/2}$$

$$V_1(t) = \frac{g^4}{64\pi M^2(4\mu^2-t)} \tag{8}$$

with $t = -(\vec{p} - \vec{p}')^2$ being the momentum transfer. The conclusions are that (i) the leading term V_o cancels for all choices of ν, but that (ii) only the choices $|\nu| = 1$ give an energy independent, local potential to order M^{-1}. Note that while $\nu = 0$ minimizes the local piece of the potential, it also maximizes the non-local, energy dependent piece. For the realistic chiral model, the 4 terms are

$$
\begin{aligned}
V_{4S} &= (3-2\tau_1\cdot\tau_2)\{-U_o + \tfrac{\mu}{M}(1-\nu^2)U_1 + \tfrac{\mu}{M}(1-\nu^2)\theta U_2\} \\
V_{4-} &= (3+2\tau_1\cdot\tau_2)\{-U_o + \tfrac{2\mu}{M} U_1\} \\
V_{4\Delta} &= 12\, U_o - \tfrac{12\mu}{M} U_1 \\
V_{4B} &= -6U_o
\end{aligned}
\qquad (9)
$$

where θ is as defined in (7), and

$$
\begin{aligned}
U_o &= \frac{g^4}{64\pi^2 M^2} \int_{4\mu^2}^{\infty} \frac{d\xi}{\xi - t} \left\{\frac{[\xi - 4\mu^2]}{\xi}\right\}^{1/2} \\
U_1 &= \frac{g^4}{64\pi^2 M^2} \frac{\pi}{4} \int_{4\mu^2}^{\infty} \frac{d\xi}{\xi - t} \frac{\xi - 2\mu^2}{\mu\sqrt{\xi}} \\
U_2 &= \frac{g^4}{64\pi^2 M^2} \pi \int_{4\mu^2}^{\infty} \frac{d\xi}{\xi - t} \frac{\mu}{\sqrt{\xi}}
\end{aligned}
\qquad (10)
$$

The conclusions are the same; the leading term U_o cancels for all ν, but only for $|\nu| = 1$ does the energy dependent, non-local part vanish.

2. IMPLICATIONS FOR PROTON-NUCLEUS SCATTERING

Before turning to the applications of these ideas to the few body systems, it is interesting to see how they provide a natural foundation for the Dirac treatment of proton-nucleus scattering reviewed elsewhere at this conference.

If the target is a heavy closed shell nucleus, and its internal structure is ignored, then proton-nucleus scattering is precisely the case described in Section 1 and illustrated schematically in Fig. 4. We have a light spin 1/2 particle (the proton) scattering from a massive spin zero, I=0, particle (the nucleus). If the basic interaction between the proton and the nucleus can be described by meson exchange, then the discussion following Fig. 4 shows that the scattering can be described by a Dirac equation for the proton moving under the influence of an instantaneous OBE potential generated by the nucleus. The OBE potential is exact in the limit when A (the mass number of the nucleon) becomes very large. The fact that the nucleus has spin and isospin zero limits the form of the OBE potential to I=0 pieces of a scalar and the 4th component of a 4 vector, in agreement with the treatment described elsewhere in these proceedings.

The structure of the nucleus modifies this elementary point of view in two ways. First, the structure of the meson-nucleus vertex must be taken into account, and the simplest, relativistic impulse approximation for this vertex is the triangle diagram shown in Fig. 6, where the (A-1) nuclear subsystem, which is a spectator to the meson-nucleon interaction, is taken to be on-shell. (This approximation has been used for the deuteron form factor, where A = 2 and the meson is replaced by the virtual photon exchanged between the deuteron and the scattered electron.[18]) One must sum over all nuclear states of the (A-1) spectator system. The interaction nucleon is off-shell, but it is an excellent approximation to retain its on-shell degrees of freedom only, except possibly at very high momentum transfer. The nucleon - A - (A-1) vertex labeled Γ in the figure, can be related to the single particle (relativistic) nucleon wave function, and the triangle diagram is the relativistic analogue of the single particle density.

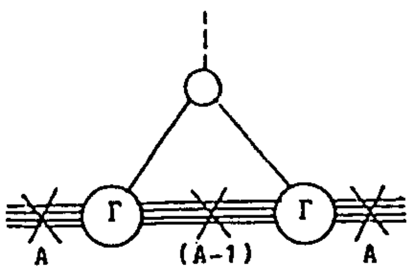


FIGURE 6

A second consequence of the structure of the nucleus which must be taken into account is the presence of excited states of the target nucleus, which

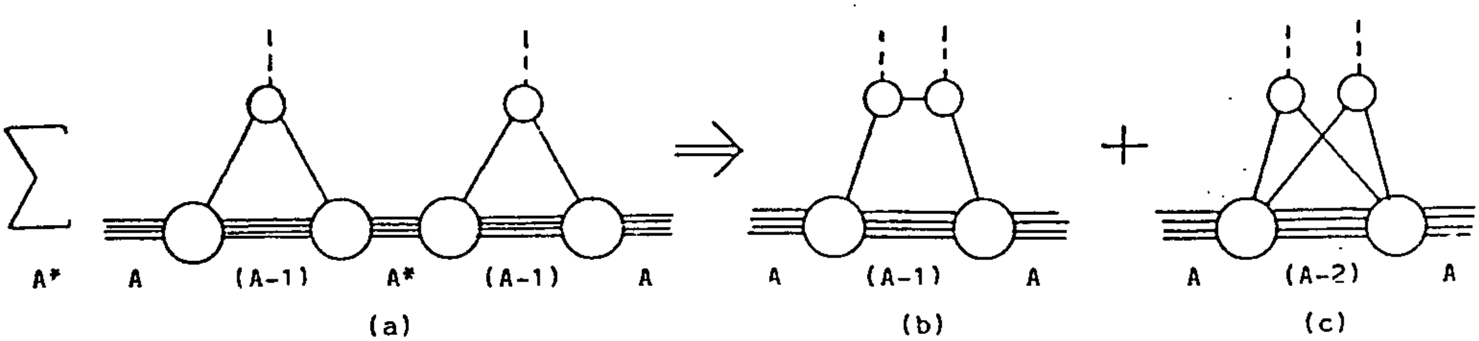


FIGURE 7

will be denoted A*. This means that the second order interaction will contain a sum over A* states, as shown in Figure 7a. If we replace this sum by free proton - (A-1) scattering we obtain the diagram shown in Fig. 7b. (Other contributions, such as those shown in Fig. 7c, will be neglected.) Doing this to all orders, and recalling the integral equation for the nucleon-nucleon scattering amplitude M, these diagrams reduce to the more general triangle diagram shown in Fig. 8. [This diagram is the basis of what is referred to as the relativistic impulse approximation (RIA) in the proton-nucleus field. The simpler diagram, Fig. 6, where the meson is replaced by a photon and the off-shell character of the interacting nucleon is retained, is referred to as the RIA in the field of electromagnetic interactions.] This then includes all iterations and interactions <u>except</u> those in which the target nucleus is in its ground state. Fig. 8 is therefore the lowest order optical potential - and when inserted in the Dirac equation gives the full result, including successive interactions in which the target nucleus remains in its ground state.

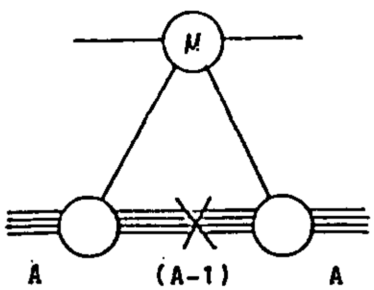


FIGURE 8

The "derivation" given above is oversimplified; interactions of the kind shown in Fig. 7c have been neglected, and these give rise to the higher order corrections to the optical potential. Nevertheless, the discussion shows how relativistic meson theory permits the consistent treatment of both the few body system and proton-nucleus scattering, and shows how they are interconnected.

3. APPLICATIONS OF RELATIVISTIC FEW BODY EQUATIONS

I now want to discuss a few applications of these ideas to calculations of the bound state and scattering properties of few body systems.

3.1 The Two Nucleon System

Fits to the two nucleon phase shifts for energies below 300 MeV have been obtained by Fleischer and Tjon[19] and Zuilhof and Tjon[20] using the BS equation in OBE approximation. These fits have been extended to energies up to 1000 MeV by van Faasen and Tjon[21], who describe the inelasticity by including NΔ intermediate states. Fits to the phase shifts below 400 MeV have also been obtained by Gross and Holinde using the G_1 equation[22], and I want to describe these new, unpublished results in a little more detail.

The relativistic kernel employed in Ref. (22) consists of an OBE model with only four mesons: π, σ, ρ, and ω. (Instead of varying the σ mass and coupling constant, two sigmas of fixed masses at 350 MeV and 760 MeV were chosen, and the couplings of each varied.) Form factors were used at the meson-nucleon vertices, and a form factor was also used with the off-shell nucleon propagator to improve convergence. While only four mesons are used, the number of parameters varied is similar to that used in conventional OBE models with more mesons, because two mixing parameters were used which do not appear in usual approaches. These are λ and μ, where λ varies the fraction of γ^5 to $\gamma^5\gamma^\mu$ coupling at the πNN vertex[1], and μ varies the fraction of $\sigma^{\mu\nu}$ and P^μ couplings at the ρNN vertex. The πNN and ρNN couplings were defined so that when the nucleons are on their mass shell the coupling is strictly independent of the value of the mixing parameter (for the ρNN coupling one uses the Gordon decomposition, which only holds on shell, to transform $\sigma^{\mu\nu}$ into P^μ). Hence, dependence of the results on these two parameters is a direct measure of the possible importance of off shell effects, and we find that such effects are large. In fact it is because of the splitting between the $^1S_0$ and $^3S_1$ phase shifts introduced by the λ dependence that we do not need the isovector-scalar meson δ in these fits.

Another novel feature of the G_1 equation employed in these fits is that the off-shell Dirac nucleon has four spin states; two for its positive energy state and two for its negative energy state. One can separate the positive and negative energy "channels", giving a coupled set of equations of the following form[1]

$$\begin{aligned}(2E_k - W)\,\psi^+ &= V^{++}\psi^+ + V^{+-}\psi^- \\ W\,\psi^- &= V^{-+}\psi^+ + V^{--}\psi^-\end{aligned} \qquad (11)$$

In Ref. (22) the approximation $V^{--} = 0$ was taken, yielding the "solution"

$$(2E_k - W)\psi^+ = \left(V^{++} + \frac{|V^{+-}|^2}{W}\right)\psi^+ \qquad (12)$$

which shows how the negative energy channel, which makes no contribution to the asymptotic states, modifies the effective interaction at short range. To obtain Eq. (12) one uses the fact that the matrix potential in Eq. (11) is hermitian. Energy dependent, effective interactions of the type shown in Eq. (12) are also familiar from the study of proton-nucleus scattering.

Another novel feature of this treatment is the presence of virtual "wrong" symmetry channels. These channels, which are symmetric under the interchange of three momentum and spin indices, are not forbidden by the Pauli principle in a region of phase space where the relative energy p_o is not zero, but must vanish when $p_o = 0$ (i.e. when both particles are on shell). It turns out that it is necessary to explicitly antisymmetrize the potential to guarantee that these channels are really virtual (i.e. are zero when $p_o = 0$), and this has been done in Ref. (22). These effects are present only for partial waves where L=J.

The result of all these considerations is that the coupled equations (11) contain four channels for all partial waves. For partial waves which are coupled by the tensor force in non-relativistic theory (e.g. $^3S_1 - {}^3D_1$) there is a doubling due to the presence of negative energy states. For partial waves which were formally uncoupled, there is both a doubling due to the presence of negative energy states, and due to coupling to virtual wrong symmetry channels.

Relativistic and off mass shell effects can then be investigated. It has been known for over ten years that a major effect of the negative energy channel is to provide repulsion at short range[1]. Fig. 9, taken from Ref. (1), shows that the quadratic terms, which are the squared terms in the effective potential given in Eq. (12), are large and repulsive. We find the same effect in the actual fits to the phase shifts, so that these effects are not an artifact of the approximations made in Ref. (1). Preliminary results give a value of $\lambda = 0.331$, very close to the value of $\lambda = 0.41$ preferred in Ref. (1), and show that V^{+-} terms coming from mesons other than the pion are, in some cases of comparable importance to the large V_π^{+-} contribution. One effect which is probably due to this repulsion is that the ω coupling constant which emerges from this fit is

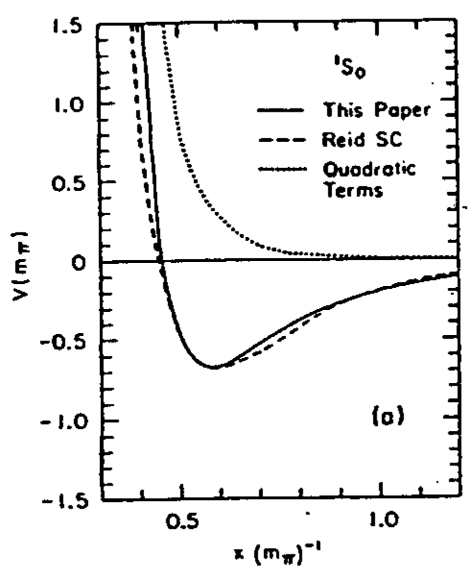


FIGURE 9

$$\frac{g_\omega^2}{4\pi} = 9.52$$

a value similar to that obtained in Ref. (20), and considerably smaller than that needed in many non-relativistic OBE fits.

The effect of the coupling to virtual wrong symmetry channels has been looked at for the $^1P_1$, $^3P_1$, and $^3D_2$ channels. In all of these cases the effects at 300-400 MeV are well outside of the error bars for the empirical phase shifts; in the $^1P_1$ case this coupling alters the shape of the phase shifts in a helpful way, and in the $^3D_2$ case it provides helpful repulsion.

Deuteron wave functions have not yet been obtained for this case. However, it seems very likely that the wave functions obtained in Ref. (25) and shown in Fig. 10 will be very similar to what will be obtained here. In addition to the large S and D state wave functions u and w, one obtains smaller negative energy P state wave function v_t and v_s corresponding to spin singlet and triplet combinations. Relativistic deuteron wave functions have also been obtained by Zuilhof and Tjon[26].

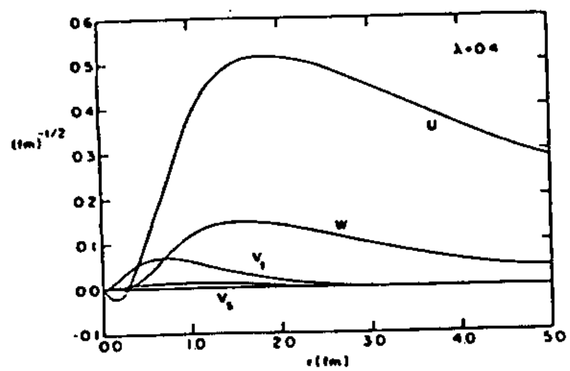


FIGURE 10

What is one to conclude from all this? While it is somewhat early to say, it is my view that almost any equation with a sufficient number of bosons and about 10 parameters can be made to fit the NN phase shifts below 400 MeV. This does not mean that the differences between relativistic equations are small, or that the relativistic effects themselves are small. In fact, such differences are known to be numerically large[20,27]. Rather, it appears that adjustments of 10 parameters can largely compensate for these differences. Since the parameters have physical significance, the extent to which their adjusted values agree with values determined from other physical processes could be a test of the validity of the equation. Perhaps a better method is to see how well a given equation, "tuned" to the two body problem,is able to describe the three body problem, nuclear matter, and other calculable systems, such as proton-nucleus scattering.

3.2 Other Systems

There is evidence that relativistic effects increase the binding of the three body system, reducing the discrepancy between the calculations and observed binding. Coester and Wiringa[28] found an increase of 1.7 MeV for the triton binding and 4.3 MeV for the alpha binding, and Rupp[29], using a separable BS equation, found similar effects. Unfortunately, neither calculation can be regarded as treating the dynamics in a realistic way. A fully relativistic treatment of the three body system, with realistic dynamics consistent with the two body problem, is needed. Such a calculation, using the three body version of the G_1 equation is possible[14]. This approach satisfies the cluster property, yields relativistic Faddeev equations with the same number of momentum variables as the non-relativistic equation, and (as in the non-relativistic case) can be reduced analytically to a two dimensional integral equation for coupled partial waves[30].

REFERENCES

1. F. Gross, Phys. Rev. D 10(1974) 223.
2. E. E. Salpeter and H. A. Bethe, Phys. Rev. 84(1951) 1232.
3. F. Gross, Phys. Rev. 186(1969) 1448.
4. A. A. Logunov and A. N. Tavkhelidze, Nuovo. Cim. 29(1963) 380.
5. R. Blankenbecler and R. Sugar, Phys. Rev. 142(1966) 1051.
6. I. T. Todorov, Phys. Rev. D 10(1971) 2351.
7. S. Weinberg, Phys. Rev. 150(1966) 1313.
8. For applications of these ideas to QED, see for example S. Brodsky, R. Roskies and R. Suaya, Phys. Rev. D 8(1973) 4574.
9. J. M. Namyslowski, Proceedings of the Graz Conference (1978) Lecture Notes in Physics #82,Ed. by Zingl et al., (Springer-Verlag) p. 41.
10. G. P. Lepage and S. J. Brodsky, Phys. Rev. D 22(1980) 2157.
11. M. Chemtob, Nucl. Phys. A336(1980) 299.
12. L. L. Frankfurt and M. I. Strikman, Nucl. Phys. B. 148(1979) 107, Physics Reports 76(1981) 215.
13. J. M. Namyslowski and H. J. Weber, Zeit. fur. Physick A295(1980) 219.
14. F. Gross, Phys. Rev. C 26(1982)2226.
15. F. Gross, Phys. Rev. C 26(1982) 2203.
16. A. R. Neghabian and W. Glockle, Can. J. of Phys. 61(1983) 85.
17. G. J. M. Austen and J. J. de Swart, Phys. Rev. Letters 50(1983) 2039.
18. R. Arnold, C. Carlson and F. Gross, Phys. Rev. C 21(1980) 1426.
19. J. Fleischer and J. A. Tjon, Phys. Rev. D 21(1980) 87.
20. M. J. Zuilhof and J. A. Tjon, Phys. Rev. C 24(1981) 736.
21. E. E. van Faassen & J. A. Tjon, Maryland preprint ORO 5126-209(1984).
22. F. Gross and K. Holinde, in preparation.
23. M. Lacombe, et al, Phys. Rev. C 21(1980) 861.
24. K. Holinde, private communication.

25. W. W. Buck and F. Gross, Phys. Rev. D 20(1979) 2361.

26. M. J. Zuilhof and J. A. Tjon, Phys. Rev. C 22(1980) 2369.

27. L. Muller and W. Glockle, Nucl. Phys. B 146(1978) 393.

28. F. Coester and R. B. Wiringa, Proceedings of the Karlsruhe Conference (1983), Few Body Problems in Physics, Vol. II, Ed. by B. Zeitnitz (North Holland) p. 343.

29. G. Rupp, contribution to the IX European Conference on Few Body Problems in Physics, Tbilisi, USSR, Aug. 25-31, 1984.

30. A. Bulla, F. Gross, H. Hennings, and P. Sauer - in preparation.